Short communication

Shear rate in stirred tank and bubble column bioreactors

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Abstract

A rigorous theoretical analysis is used to show that for both Newtonian and non-Newtonian power law fluids agitated in stirred vessels, the average shear rate γ in the fluid is a function of the rotational speed N of the impeller, as follows:

$$\gamma = \text{constant} \cdot N \text{ (laminar flow)}$$

$$\gamma = \text{constant} \cdot N^{3/2} \text{ (turbulent flow).}$$

Only in turbulent flow, the proportionality constant in the above equation depends on the flow index and the consistency index of the power law fluid. The above equations derived by theoretical reasoning are in excellent agreement with the long established empirical art.

In bubble columns, the average shear rate depends on the superficial gas velocity U_g, as follows:

$$\gamma = \text{constant} \cdot U_g^{1/(n+1)}$$

where n is the flow index of the power law fluid. The proportionality constant in the above equation for bubble columns is a function of the flow index, consistency index and the density of the liquid.
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1. Introduction

Stirred tanks and bubble columns are widely used as mixing vessels and bioreactors. Average spatial fluid velocity gradients at the level of the eddies or the prevailing shear rate, is an important variable in bioreactors but is not easy to characterize. A knowledge of shear rate is essential for at least two main reasons: (1) shear rate influences the average apparent viscosity of non-Newtonian fluids and hence affects power absorption, mixing characteristics and mass transfer phenomena [1]; (2) microorganisms, bioflocs and other suspended solids are susceptible to damage that is dependent on the prevailing shear rate and associated shear stress [2,3].

The main equations for estimating average shear rate γ and the maximum shear rate γ_{max} in the impeller zone of stirred tanks are summarized in Table 1 [2,3]. Most authors have correlated

shear rate with the rotational speed of the impeller [4–7], or with power input that depends on impeller speed [8,9]. The equations in Table 1 were all obtained empirically. Here we show by purely theoretical reasoning that the average shear rate in Newtonian and non-Newtonian media in a stirred vessel is a function of only the rotational speed N (laminar flow), or $N^{3/2}$ (turbulent flow). As demonstrated further, these theoretical outcomes are in excellent agreement with the well-established prior art.

In bubble columns, the sole source of agitation is the pneumatic power input provided by isothermal expansion of the sparged gas. For such cases, the average shear rate is shown to exclusively depend on the superficial gas velocity and the rheological properties of the fluid in ways that are consistent with other independent theoretical analyses [10,11].

2. Theory

The specific energy dissipation rate in a stirred tank is well known to depend on the shear rate γ and the shear stress τ [12],

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Nomenclature

a	gas–liquid interfacial area per unit volume of liquid in bubble column (m^{-1})
C	constant in Eq. (9)
d_i	diameter of the impeller (m)
d_T	diameter of tank (m)
H	height of fluid in tank (m)
k_i	impeller constant
K	consistency index $(Pa\,s^n)$
m	constant in Eq. (24)
M	torque (N m)
n	flow index
N	agitation speed (s^{-1})
N_p	Power number
P	power input (W)
q	constant in Eq. (20)
Re	impeller Reynolds number
V	volume of fluid (m^3)
V_s	volume swept by the impeller (m^3)
w	constant in Eq. (23)
W	width of impeller blade (m)

Greek symbols

ε	energy input per unit mass $(W\,kg^{-1})$
γ	average shear rate (s^{-1})
γ_{max}	maximum shear rate (s^{-1})
μ	viscosity (Pa s)
μ_a	apparent viscosity (Pa s)
π	the number pi
ρ	density of fluid $(kg\,m^{-3})$
τ	shear stress (Pa)
Ψ	constant in Eq. (29)

Table 1
Correlations for shear rate in stirred tanks

Equation	Reference
$\gamma = k_i N$	Metzner and Otto [4]
$\gamma = 4.2N\left(\frac{d_i}{d_T}\right)^{0.3}\frac{d_i}{W}$	Bowen [6]
$\gamma = k_i\left(\frac{4n}{3n+1}\right)^{n/(n-1)}N$	Calderbank and Moo-Young [5]
$\gamma = \frac{0.367}{\mu}\left(\frac{P}{V}\left(\frac{V}{V_s N_p}\right)^{0.42}\right)^{0.55}$	Hoffmann et al. [8]
$\gamma = \left(\frac{P}{V\mu_a}\right)^{0.5}$	Henzler and Kauling [10] cited by Candia and Deckwer [9]
$\gamma_{max} = 9.7N\left(\frac{d_i}{d_T}\right)^{0.3}\frac{d_i}{W}$	Bowen [6]
$\gamma_{max} = 3.3N^{1.5}d_i\left(\frac{\rho}{\mu}\right)^{0.5}$	Robertson and Ulbrecht [7]
$\gamma_{max} = N(1+5.3n)^{1/n}\left(\frac{N^{2-n}d_i^2\rho}{K}\right)^{1/(1+n)}$	Wichterle et al. cited in Robertson and Ulbrecht [7]

as follows:

$$\frac{P}{V} = \tau\gamma \tag{1}$$

where P is the power input and V is the volume of the fluid in the tank. Furthermore, for Newtonian fluids, the viscosity μ is the ratio of shear stress and shear rate, i.e.

$$\mu = \frac{\tau}{\gamma}, \tag{2}$$

therefore, Eq. (1) can be written as follows:

$$\frac{P}{V} = \tau\gamma = \tau\gamma\frac{\gamma}{\gamma} = \mu\gamma^2, \tag{3}$$

or

$$\gamma = \left(\frac{1}{\mu}\frac{P}{V}\right)^{1/2}. \tag{4}$$

Eq. (4) applies to laminar, turbulent and transitional flows.

For non-Newtonian fluids obeying the power law [1], we have:

$$\tau = K\gamma^n \tag{5}$$

where K is the consistency index and n is the flow behavior index of the fluid. Because the apparent viscosity μ_a is given as follows [4]:

$$\mu_a = \frac{\tau}{\gamma} = K\gamma^{n-1}, \tag{6}$$

for non-Newtonian media the equation corresponding to Eq. (3) becomes the following:

$$\frac{P}{V} = \mu_a\gamma^2 \tag{7}$$

and, therefore,

$$\gamma = \left(\frac{1}{K}\frac{P}{V}\right)^{1/(n+1)}. \tag{8}$$

Eq. (8) applies to both laminar and turbulent flow regimes.

For agitation under laminar flow ($Re \leq 10$), the Power number (N_p) and the agitator Reynolds number (Re) are related [13,14] as follows:

$$N_p = \frac{C}{Re} \tag{9}$$

where the constant C depends on the geometry of the tank and the impeller [13,14]. Substituting the following definitions [1] of the Power number and Reynolds number,

$$N_p = \frac{P}{\rho N^3 d_i^5} \tag{10}$$

$$Re = \frac{\rho N d_i^2}{\mu} \tag{11}$$

in Eq. (9), we obtain the following equation:

$$\frac{P}{\rho N^3 d_i^5} = C\left(\frac{\mu}{\rho N d_i^2}\right). \tag{12}$$

In Eqs. (10)–(12), ρ is the density of the fluid, N the rotational speed of the agitator and d_i is the diameter of the impeller.

The power input of course depends on the torque M on the impeller, as follows:

$$P = 2\pi M N. \tag{13}$$

For Newtonian fluids, the substitution of Eq. (13) in Eq. (12) leads to the following equation:

$$M = \frac{C\mu d_i^3}{2\pi} N. \tag{14}$$

Eq. (14) applies to laminar flow.

In turbulent flow, the Power number is constant [13,14], therefore, from Eqs. (10) and (13) we have the following:

$$N_p = \frac{P}{\rho N^3 d_i^5} = \frac{2\pi N M}{\rho N^3 d_i^5} \tag{15}$$

or

$$M = \frac{N_p d_i^5 \rho}{2\pi} N^2. \tag{16}$$

From Eq. (13) for a stirred tank of standard geometry [1], i.e. $H = d_T = 3\, d_i$, the specific power input can be expressed as follows:

$$\frac{P}{V} = \frac{2\pi N M}{\pi/4\,(3d_i)^3} = \frac{8}{3^3 d_i^3} N M. \tag{17}$$

Substitution of Eq. (14) in Eq. (17) gives the equation:

$$\frac{P}{V} = \frac{8}{3^3 d_i^3} N \frac{\mu C d_i^3}{2\pi} N = \frac{4\mu C}{\pi 3^3} N^2. \tag{18}$$

Hence, in laminar flow, the specific power input is a function of N^2. Eqs. (3) and (18) imply that

$$\frac{P}{V} = \mu\gamma^2 = \frac{4\mu C}{\pi 3^3} N^2 \tag{19}$$

or

$$\gamma = \left(\frac{4C}{\pi 3^3}\right)^{1/2} N = qN \tag{20}$$

where q is a constant. Eq. (20) is valid for both Newtonian and non-Newtonian fluids.

For turbulent flow ($Re > 10^4$) in standard stirred tanks, Eqs. (16) and (17) lead to the following equation:

$$\frac{P}{V} = \frac{8}{3^3 d_i^3} N \frac{N_p \rho d_i^5}{2\pi} N^2 = \frac{4N_p \rho d_i^2}{\pi 3^3} N^3. \tag{21}$$

Thus, in turbulent flow, the specific power input is a function of N^3. Furthermore, from Eqs. (3) and (21), we can deduce the following:

$$\frac{P}{V} = \mu\gamma^2 = \frac{4N_p \rho d_i^2}{\pi 3^3} N^3, \tag{22}$$

or

$$\gamma = \left(\frac{4N_p \rho d_i^2}{\pi 3^3 \mu}\right)^{1/2} N^{3/2} = wN^{3/2} \tag{23}$$

where w is a constant for a Newtonian fluid because the Power number and viscosity are constants.

For turbulent flow in non-Newtonian fluids, replacing the viscosity term in Eq. (23) by Eq. (5) and further rearrangement can be used to obtain the following equation for average shear rate:

$$\gamma = \left(\frac{4N_p \rho d_i^2}{\pi 3^3 K}\right)^{1/(1+n)} N^{3/(1+n)} = mN^{3/(1+n)} \tag{24}$$

where m is a constant. Eq. (24) is quite general and applies also to Newtonian media ($n = 1$, $K = \mu$).

2.1.1. Extension to bubble columns

In bubble columns, the power input per unit volume of liquid is related with the superficial gas velocity U_g, as follows:

$$\frac{P}{V} = g\rho U_g \tag{25}$$

Substitution of the above in Eq. (8) leads to the following expression:

$$\gamma = \left(\frac{1}{K} g\rho U_g\right)^{1/(n+1)}. \tag{26}$$

Eq. (26) is identical to the equation that was obtained for bubble columns by Henzler and Kauling [10] through dimensional analysis. Henzler and Kauling [10] related the average shear rate in bubble columns to the energy input per unit mass (i.e. ε), as follows:

$$\gamma = \left(\frac{\rho\varepsilon}{K}\right)^{1/(n+1)} \tag{27}$$

where

$$\varepsilon = gU_g. \tag{28}$$

An equation similar to Eq. (26) was derived theoretically by Kawase and Kumagai [11]. This equation included a proportionality constant Ψ as follows:

$$\gamma = \Psi^{2/n} \varepsilon^{1/(n+1)} \left(\frac{K}{\rho}\right)^{-1/(n+1)}. \tag{29}$$

If in Eq. (29) the value of Ψ is estimated as proposed by Kawase and Moo-Young [15] and Eq. (28) is substituted for the specific energy input, the following equation is obtained:

$$\gamma = (10.3n^{-0.63})^{1/(n+1)} (U_g g)^{1/(n+1)} \left(\frac{K}{\rho}\right)^{-1/(n+1)}. \tag{30}$$

Except for the multiplier $(10.3n^{-0.63})^{1/(n+1)}$, Eq. (30) is identical to Eqs. (26) and (27).

3. Discussion

The theoretically derived Eqs. (20) and (23) suggest that in Newtonian media, the average shear rate depends on N

and $N^{3/2}$ in laminar ($Re < 10$) and turbulent ($Re > 10^4$) flows, respectively. This theoretically derived dependence is of course in complete agreement with the well-established empirical equation of Metzner and Otto [4] (Table 1), thus revealing a previously unknown theoretical foundation for that equation. Other empirical correlations that suggest a direct dependence between shear rate and N are those due to Bowen [6] and Calderbank and Moo-Young [5], as shown in Table 1. Clearly, the empirical evidence is overwhelmingly consistent with the theory.

In turbulent flow, the empirical evidence shows that the shear rate depends on $N^{1.5}$ (Table 1; [7]) and this is consistent with the theoretically derived Eq. (23). As a further empirical evidence for Eq. (23), Kelly and Gigas [17] correlated values of the average impeller shear rate obtained through computational fluid dynamic modelling, with the rotational speed of the impeller, as follows:

$$\gamma = 64.3N. \tag{31}$$

The regression coefficient for the above equation was 0.96 [17]; however, for the same data (Fig. 10a of reference [17]), we obtained the following correlation:

$$\gamma = 33.1N^{1.4}. \tag{32}$$

Eq. (32) correlated the data with a regression coefficient of 0.999, or substantially better than Eq. (31) proposed by Kelly and Gigas [17]. The exponent on N-term in Eq. (32) is quite close to the theoretically derived value of 1.5. Eqs. (31) and (32) are for an A315 axial flow hydrofoil impeller (LIGHTNIN Mixers, Rochester, NY, USA) operated in the transitional flow regime.

As further evidence in support of the analysis presented here, for non-Newtonian power law fluids, the empirical equation cited by Robertson and Ulbrecht [7] (Table 1) reveals that the shear rate is proportional to $N^{3/1+n}$, or in exact agreement with the theoretical Eq. (24). In non-Newtonian media, as expected [7,16], the shear rate depends on the rheological properties of the fluid in addition to depending on the rotational speed. Clearly, empirical evidence from three independent sources [7, Wichterle et al. cited in 7, 17] supports the theoretical Eqs. (23) and (24).

In bubble columns, the mechanistically derived Eq. (26) shows that the average shear rate depends on the superficial aeration velocity, the rheological properties of the power law fluid and its density. This mechanistically derived relationship is supported by independent analyses [10,11]. Although many empirical correlations for shear rate in bubble columns have disregarded any relationship between shear rate and properties of the fluid (i.e. K, n and ρ) (see Chisti [2] for a review), this approach has been questioned in the literature [2,16,18].

In summary, the agreement of the theory discussed here with several independent empirical observations in different flow regimes and types of fluids, strongly supports the theoretical Eqs. (20), (23), (24) and (26).

4. Conclusions

Theoretical reasoning presented here leads to equations for correlating the average shear rate in stirred vessels operated with various types of fluids in laminar and turbulent flow regimes. The theoretically derived Eqs. (20), (23) and (24) are overwhelmingly consistent with well-known independent empirical observations, thus lending credence to the theoretical reasoning used and rationalizing the prior empirical observations. In both Newtonian and power law fluids, the average shear rate in laminar flow is confirmed to depend on the impeller rotational speed N. In turbulent flow in both Newtonian and non-Newtonian media, the average shear rate is shown to depend on $N^{3/1+n}$. In bubble columns, the average shear rate is related with the superficial aeration velocity and the rheological properties of the fluid in accordance with Eq. (26). This mechanistic relationship agrees with other independent observations.

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